A QUALITATIVE INVESTIGATION OF THE EQUATIONS OF QUASI-ONE-DIMENSIONAL MAGNETOHYDRODYNAMIC CHANNEL FLOW

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A qualitative investigation of the system of differential equations describing the quasi-one-dimensional flow of an electrically conducting medium at small magnetic Reynolds numbers gives an idea of the different possible flow patterns occuring when the electromagnetic field and channel shape are given in different ways. Such a treatment is essential for the calculation of one-dimensional flows, and also for the solution of variational problems [1].

In the literature devoted to this question studies have been made of flow in a one-dimensional electromagnetic field and a channel of constant cross section [2], as well as of the flow when the magnetic field is described by specially given functions of the flow velocity [3]. These cases reduce to the analysis of integral curves in a plane.

In the present paper the investigation is carried out for an arbitrary distribution of the electric and magnetic fields and channel shape, which leads to a consideration of the behavior of integral curves in three-dimensional space. The qualitative results are illustrated by examples.

1. We consider the steady-state flow of an ideal, compressible, electrically conducting medium in a flat channel (Fig. 1) with an external magnetic field $B^\circ = (0, 0, -B^\circ)$. The upper and lower walls of the channels are conductors with potentials φ° and $-\varphi^\circ$, respectively. For $x^\circ < 0$ the channel walls are insulators and $B^\circ \equiv 0$. The gas flows in the positive direction of the x axis from a reservoir, where it has a density $\rho_s{}^\circ$, enthalpy $h_s{}^\circ$ and electrical conductivity $\sigma_s{}^\circ$.

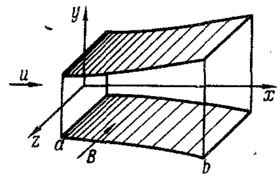


Fig. 1

Assuming that the flow is one-dimensional, that the magnetic Reynolds number is small, that the usual form of Ohm's law is valid, and supposing that the medium is a perfect gas, we write down the equations of motion, energy and continuity [1],

$$\rho uu' + p' + \Delta\sigma B\left(uB - \frac{\varphi}{y}\right) = 0 \qquad \left(\Delta = \frac{B_*{}^{\circ 2}\sigma_s{}^\circ l^\circ}{\rho_s{}^\circ \sqrt{2h_s{}^\circ}}\right),$$

$$\left[yu\left(\frac{\varkappa}{\varkappa - 1}p + \frac{\rho u^2}{2}\right)\right]' + \Delta\sigma\varphi\left(uB - \frac{\varphi}{y}\right) = 0,$$

$$\rho uy = m. \qquad (1.1)$$

Here 2y is the height of the channel, u is the velocity, p is the pressure, m the flowrate, Δ is a dimensionless parameter, $\varkappa$ is the ratio of specific heats, a prime indicates derivatives with respect to x, the quantities with the index ° have dimensions, those without it are dimensionless. The relation between dimensional and dimensionless variables is given by the relations

$$x = \frac{x^\circ}{l^\circ}, \quad y = \frac{y^\circ}{y_a{}^\circ}, \quad u = \frac{u^\circ}{\sqrt{2h_s{}^\circ}}, \quad \rho = \frac{\rho^\circ}{\rho_s{}^\circ},$$

$$p = \frac{p^\circ}{2\rho_s{}^\circ h_s{}^\circ}, \quad \sigma = \frac{\sigma^\circ}{\sigma_s{}^\circ}, \quad B = \frac{B^\circ}{B_*{}^\circ}, \quad \varphi = \frac{\varphi^\circ}{y_a{}^\circ B_*{}^\circ \sqrt{2h_s{}^\circ}},$$

where l° is the length of the channel, B_*° is the maximum magnetic field strength for $0 \le x \le 1$, the subscripts a, b are assigned to parameters in the initial and final cross sections of the channel, the subscript s refers to parameters in the reservoir.

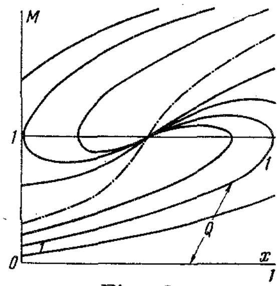


Fig. 2

It follows from (1.1) that flow in the channel is determined by its shape y(x), the magnetic field strength B(x), the electric potential $\varphi(x)$, and the boundary conditions in the initial and final cross sections of the channel. We shall assume that the functions B(x) and $\varphi(x)$ are continuous everywhere with the exception of isolated points where they may have a discontinuity of the first kind, and y(x) is continuous and positive.

We shall further assume that for x = 0 the parameters u_a, p_a, ρ_a are known, and for x = 1 they fulfill one of the conditions

$$M_b < 1, \quad p_b = p_\infty, \qquad \left(M = \sqrt{\frac{mu}{\varkappa py}}\right), \qquad (1.2)$$

$$M_b \geqslant 1, \quad p_b \geqslant p_\infty, \qquad (1.3)$$

depending on the flow configuration, where M_b is the Mach number at the exit cross section of the channel, and p_∞ is the pressure of the medium which the flow enters.

Introducing the Mach number M in place of the pressure p, and eliminating ρ with the help of the continuity equation, we obtain, instead of (1.1), the equivalent system

$$M' = \frac{M\left[1 + {}^1/_2(\varkappa - 1)M^2\right]}{mu^2(1 - M^2)}\left(\varkappa M^2 y\Delta\sigma\alpha\beta_1 - mu^2\frac{y'}{y}\right),$$

$$u' = \frac{u}{mu^2(1 - M^2)}\left(\varkappa M^2 y\Delta\sigma\alpha\beta_2 - mu^2\frac{y'}{y}\right). \qquad (1.4)$$

Here

$$\alpha = uB - \frac{\varphi}{y},$$

$$\beta_1 = uB - \frac{\varkappa - 1}{\varkappa}\,\frac{1 + \varkappa M^2}{2 + (\varkappa - 1)M^2}\,\frac{\varphi}{y},$$

$$\beta_2 = uB - \frac{\varkappa - 1}{\varkappa}\,\frac{\varphi}{y}.$$

The second equation of (1.4) may be transformed to the form

$$u' = \frac{u}{M\,[1+{}^1/_2\,(\varkappa-1)\,M^2]}\,M' - \frac{f(\varkappa-1)\,\Delta\varphi M^2\alpha}{mu\,[2+(\varkappa-1)\,M^2]}. \quad (1.5)$$

For $\alpha > 0$ we have a generator configuration, for $\alpha < 0$ an accelerator configuration [2, 3]. In what follows we shall set $\sigma \equiv 1$ for simplicity in the calculations.

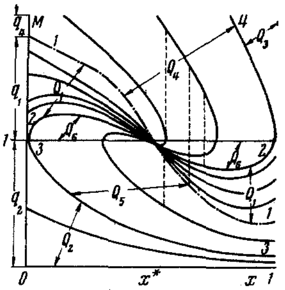


Fig. 3

2. For constant B, φ, y the system (1.4) is autonomous and may be investigated in the uM plane, as was done in [2]. For arbitrary B(x) and y(x), but $\varphi \equiv \equiv 0$ the second equation of (1.1) is integrated by quadratures and the system (1.4) reduces to the equation

$$M' = \frac{M\,[1+{}^1/_2\,(\varkappa-1)\,M^2]}{1-M^2}\quad\left(\varkappa M^2 y\Delta\frac{B^2}{m} - \frac{y'}{y}\right),$$

an analysis of which is carried out in the xM plane.

In the general case, when B, φ, y are arbitrary functions of x it is necessary to consider the space xuM, while only those integral curves belonging to the region

$$0 \leqslant x \leqslant 1, \qquad u > 0, \qquad M > 0 \qquad (2.1)$$

are of interest for the present investigation.

One of the boundaries of this region (the plane x = 0) is the plane in which the initial conditions are given, and two others, the planes u = 0 and M = 0 are composed of integral curves of system (1.4) of the form

$$M = 0, \qquad u = \text{const}\; y(x), \qquad u = 0, \qquad x = \text{const}.$$

To find the singular points, we set the numerator and denominator of the right side of Eqs. (1.4) equal to zero:

$$M\left(1+\frac{\varkappa-1}{2}\,M^2\right)\left(\varkappa M^2 y\Delta\alpha\beta_1 - mu^2\,\frac{y'}{y}\right) = 0,$$

$$u\left(\varkappa M^2 y\Delta\alpha\beta_2 - mu^2\,\frac{y'}{y}\right) = 0, \qquad mu^2\,(1-M^2) = 0\,.$$

It follows from the form of these equations that their solutions are not individual isolated points, but certain lines. These lines, which we shall call singular, are as follows:

$$M = 0, \qquad u = 0, \qquad (2.2)$$

$$M = 1, \qquad u = \left[(2\varkappa-1)\,y \pm \left(y^2 + \frac{4\,(\varkappa-1)\,my'}{\Delta B^2}\right)^{1/2}\right]\left(\frac{2\varkappa B}{\varphi}\,y^2 - \frac{2my'}{\Delta B\varphi}\right)^{-1}. \qquad (2.3)$$

We shall investigate the nature of the singularities on line (2.2), i.e., the x axis. For small u and M system (1.4) reduces to the form

$$M' = \frac{\mu_1 M^3 + \mu_2 M u^2}{mu^2}, \quad u' = \frac{2\mu_1 M^2 u + \mu_2 u^3}{mu^2} \qquad \left(\mu_1 = \frac{\Delta\varphi^2\,(\varkappa-1)}{2y},\ \mu_2 = -m\,\frac{y'}{y}\right). \quad (2.4)$$

We shall consider the behavior of the integral curves in the neighborhood of the plane u = 0, i.e., in the small region Ω determined by the inequalities $M < \varepsilon$ and $u \ll M$. Then, neglecting the second terms in the numerators of the right sides of equations (2.4), we have that $M' > 0$, $u' > 0$, since $\mu_1/m > 0$.

This means that the integral curves for $u \ll M$ emerge from the singular points of the x axis and as x increases leave the region Ω. No integral curve may enter the region Ω as x increases, and so in the neighborhood of a singular point it fulfills the relation M/u < const, which, when set in (2.4), gives $x \to \infty$ if $M \to 0$, and $u \to 0$. Thus, over a finite segment of the x axis no integral curve enters singular points on the x axis as x increases.

We shall now investigate the nature of the singular points on the line (2.3), situated in the plane M = 1.

In view of the fact that in the neighborhood of a singular point belonging to the line (2.3), an increase in u is expressed by an increase in M and x in accordance with (1.5), the integral curves in the neighborhood of this point lie in some plane, and the character of the singularity may be analyzed by the method applicable to curves in planes. The following equation is obtained for the derivatives M' calculated along the direction of curve itself:

$$M'^2 + \gamma_1 M' + \gamma_2 = 0. \qquad (2.5)$$

Here

$$\gamma_1 = \frac{\varkappa+1}{2}\,\frac{\varkappa y\Delta}{mu^2}\left(uB - \frac{\varkappa-1}{\varkappa}\,\frac{\varphi}{y}\right)\left(uB - \frac{\varkappa}{\varkappa+1}\,\frac{\varphi}{y}\right),$$

$$\gamma_2 = \gamma_2{}^* + \gamma_2{}^{**}$$

$$\gamma_2{}^* = \frac{(\varkappa+1)\,\varkappa\Delta y'}{4mu^2\beta_2}\Big[uB\beta_2\,(\alpha+\beta_2) - \frac{\varkappa-1}{\varkappa\,(\varkappa+1)}\,\frac{\varphi^2}{y^2}\left(\frac{\varkappa-1}{\varkappa}\,\alpha+\beta_2\right)\Big],$$

$$\gamma_2{}^{**} = \frac{(\varkappa+1)\,\varkappa\Delta y}{4mu^2}\Big[uB'\,(\alpha+\beta_2) - \frac{\varphi'}{y}\left(\frac{\varkappa-1}{\varkappa}\,\alpha+\beta_2\right) - \frac{y''}{y'}\,\alpha\beta_2\Big]. \qquad (2.6)$$

The singularities may have different characters [4], depending on the value of the roots of equation (2.5).

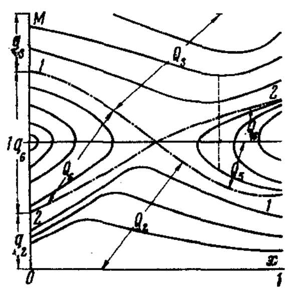


Fig. 4

First case. The roots of (2.5) are real, different in magnitude, and are both negative. In this case the singular point is a node, and in its neighborhood the coefficients in (2.6) satisfy the conditions

$$\gamma_1 > 0, \ \gamma_2 \geqslant 0, \ {}^1/_4\gamma_1^2 - \gamma_2 \geqslant 0 \cdot \qquad (2.7)$$

At such a singular point a continuous transition through the speed of sound is possible only in passing from supersonic to subsonic flow. That part of the integral curve which corresponds to supersonic flow is determined by specifying the initial conditions at $x = 0$, $M = M_a$, $u = u_a$, and the integral curve corresponding to subsonic flow from the singular point to $x = 1$ is determined by one of the conditions (1.2) or (1.3).

Second case. The roots of (2.5) are real, of different magnitude, and of opposite sign. In this case the singular point is a saddle point, and the condition

$$\gamma_2 < 0 \qquad (2.8)$$

is fulfilled in its neighborhood.

At a singularity of this type a continuous transition is possible from subsonic to supersonic flow along one of the integral curves entering the saddle point from $M' > 0$, and a continuous transition from supersonic to subsonic flow is possible along another integral curve entering the saddle point from $M' < 0$.

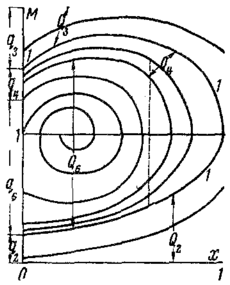


Fig. 5

An example of flow corresponding to a passage through the speed of sound at a singularity of this type is well known in gasdynamics (Laval nozzle), where the condition $y' = 0$ should be fulfilled at the transition point.

Third case. The roots of (2.5) are complex. In this case the singular point is a focus, and the condition

$${}^1/_4\gamma_1^2 - \gamma_2 < 0 \qquad (2.9)$$

is fulfilled in its neighborhood.

At a singularity of this type a continuous transition through the speed of sound is impossible. The flows which are possible in this case may be either totally supersonic, or totally subsonic, or with a transition from supersonic to subsonic configurations in a shock wave.

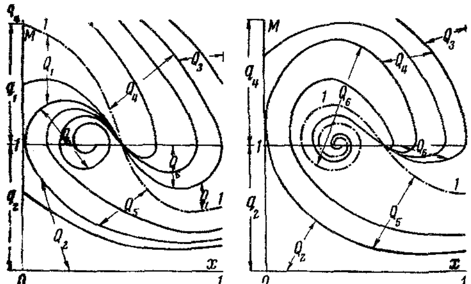
Fourth case. The roots of (2.5) are real, of different magnitude, and are both positive.

In this case the singular point is also a saddle point, and the condition

$$\gamma_1 < 0, \qquad \gamma_2 \geqslant 0, \qquad {}^1/_4\gamma_1^2 - \gamma_2 \geqslant 0 \qquad (2.10)$$

is fulfilled in its neighborhood.

Close to such a singularity a continuous transition is possible from subsonic to supersonic flow. It follows from relations (2.10), (2.3), and (2.6) that a singularity of this type may exist in a narrowing channel $y' < 0$, operating as an accelerator. Obviously a single-valued solution cannot be constructed in this case since a family of curves depending on one parameter emerges from the singular point, each of which satisfies the condition $M_b > 1$, $p_b > p_\infty$.



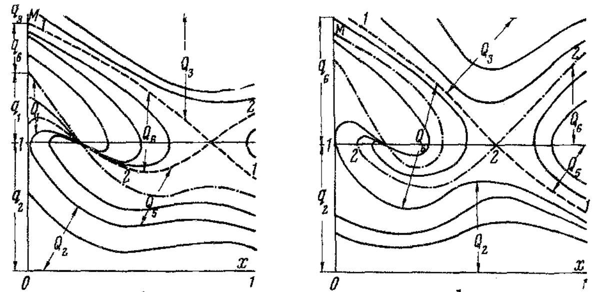
a b

Fig. 6

We note that these configurations may be obtained only by a nonstationary path, since the curves from the region Q, and its bounding curve 1, represented schematically in the plane xM in Fig. 2, correspond to a quasi-stationary increase of the pressure differential $p_s - p_\infty$. In what follows we shall not consider this type of singularity.

Different flow configurations correspond to different types of singularity. The part of the three-dimensional space under consideration (2.1) may be divided into a series of regions depending on the nature of the flow.

For the sake of clarity, we shall begin with an analysis of these regions for the simpler case when $\varphi = 0$, which reduces to an investigation of the integral curves in a plane, and we shall subsequently extend the results to the case of three-dimensional space.



a b

Fig. 7

For $\varphi = 0$, $B = B(x)$, $y = y(x)$ the coefficients (2.6) have the form

$$\gamma_1 = \frac{(\varkappa+1)\varkappa y \Delta B^2}{2m}, \qquad \gamma_2{}^* = \frac{2(\gamma_1)^2}{\varkappa+1},$$

$$\gamma_2{}^{**} = \gamma_1\left(\frac{B'}{B} - \frac{y''}{2y'}\right).$$

We shall consider the first case. * The conditions (2.7) impose the restrictions

$$-\frac{2\gamma_1}{\varkappa+1} \leqslant \frac{B'}{B} - \frac{y''}{2y'} \leqslant \frac{\varkappa-7}{4(\varkappa+1)}\gamma_1$$

*In the present paper the node type singularity is described in more detail than the others, since it corresponds to the most varied set of flow regions.

on B' and y", which when fulfilled result in a singular point which is a node (Fig. 3). The separatrix of the node 1 (it is given by a line of dots and dashes in Fig. 3) divides the integral curves entering the node for M > 1 from the curves entering the node for M < 1.

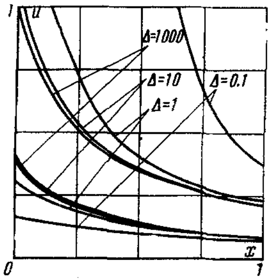


Fig. 8

Solutions with a continuous transition through the speed of sound are situated in the region Q_1 which lies between the separatrix 1 and the integral curve 2. The initial data from the segment q_1 correspond to the region Q_1 on the straight line x = 0.

The region Q_2 is situated below the integral curve 3, and its integral curves correspond to totally subsonic flow with initial conditions given on segment q_2.

Flow configurations which are totally supersonic occur in region Q_3, which is situated above integral curve 4, entering the point M = 1, x = 1. The initial data for solutions in this region are contained in the semi-infinite segment q_3.

The separatrix 1, the integral curve 4 and the straight line M = 1 divide the region Q_4, and the solution may be extended continuously along the integral curves of this region to M = 1 for x < 1. The integral curves from Q_4 correspond to supersonic flow in front of the shock wave, and the subsonic part of this solution belongs either to the region Q_2, or to the subsonic part of region Q_1, or to the region Q_5 enclosed between the integral curve 3, the straight line M = 1 and the separatrix 1.

The position of the shock wave in the channel (given in Fig. 3 by a broken line) is determined by the conditions at x = 1.

Initial data from the interval q_4 correspond to solutions with a shock wave. Integral curves from the region Q_6, bounded by the straight line M = 1 and the curve 2, do not correspond to actual flow patterns.

Certain regions may increase, decrease or vanish altogether, depending on the position of the singular point x = x* in the channel. For example, for x* = 1 the region Q_4 is absent.

We now assume that in the three-dimensional case with $\varphi(x) \not\equiv 0$ there is one singular line in the plane M = 1 composed entirely of nodes. The separatrices of the nodes from a surface in the space xuM, separating the solutions with a continuous transition through the velocity of sound from solutions with a shock wave. The separatrix surface and the plane M = 1 intersecting the plane x = 0 from a region of initial values of u, M, from which flow patterns with a continuous transition through the speed of sound commence.

The integral curves having M = 1 for x = 1 (similar to curve 4 in Fig. 3), form a surface separating supersonic flow in the channel from flow with a shock wave, and the integral curves having M = 1 for x = 0 form a surface dividing subsonic and supersonic flow from the integral curves similar to the curves from regions Q_5 and Q_6 in a plane.

On intersecting the plane x = 0 these regions define corresponding regions of initial values.

In the second case for $\varphi = 0$ conditions (2.8) give

$$\frac{B'}{B} - \frac{y''}{2y'} < -\frac{2\gamma_1}{\varkappa + 1}$$

and the singular point is a saddle point.

The separatrices 1 and 2 of the saddle point (Fig. 4) divide the field of integral curves into the regions: Q_2 below both separatrices where the flow is entirely subsonic; Q_3 above both separatrices where the flow is entirely supersonic; Q_5 between the separatrix 1 and the straight line M = 1 to the right of the singular point. The integral curves belonging to Q_5 correspond to the subsonic flow behind a shock wave if the supersonic part of the solution lies in the region Q_3 or on its border; Q_6 to the left of the singular point between the separatrices and to the right between the separatrix 2 and the straight line M = 1. The integral curves of this region do not correspond to observed flows. A continuous transition through the velocity of sound is possible only along the separatrix.

The initial data from q_2, q_3, q_6 correspond to the regions Q_2, Q_3, Q_6.

In the three-dimensional case for $\varphi = \varphi(x)$, if the singular line which occurs is composed only of saddle type singularities, the separatrices of the saddle points form a surface dividing the portion of the three-dimensional space (2.1) into regions similar to the regions in the plane, and the flow configurations which occur are similar to those considered above.

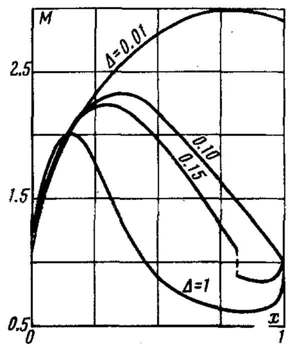


Fig. 9

For focus type singularities (Fig. 5) in the case $\varphi = 0$ the inequalities (2.9) give

$$\frac{B'}{B} - \frac{y''}{2y'} > \frac{\varkappa - 7}{4(\varkappa + 1)}\gamma_1 .$$

Here the flow may be either entirely supersonic—in the region Q_3, or entirely subsonic—in the region Q_2, or may have a transition from supersonic to subsonic flow in a shock wave.

Integral curves corresponding to the supersonic part of the solution lie in the region Q_4, and the solutions beyond the shock wave are situated in the region Q_2 and on its border—the integral curve 1.

The integral curves from the region Q_6 do not correspond to actual flow patterns.

Degenerate singularities such as centers (roots of (2.5) imaginary), nodes with one characteristic direction (roots which coincide) etc., are not analyzed here since the presence of such singularities does not lead to new types of flow.

We shall now consider different cases of interaction of the singularities for integral curves in the plane ($\varphi = 0$). For example, let there be a focus and a node in the region under consideration (Fig. 6). Two cases may occur:

Separatrix 1 of the node intersects the straight line $x = 0$, and solutions are possible with a continuous transition through the speed of sound (Fig. 6, a).

Separatrix 1 of the node does not intersect the straight line $x = 0$, and consequently there are no solutions with a continuous transition through the speed of sound (Fig. 6, b).

In the first case there will be regions of initial values q_i on the straight line $x = 0$, corresponding to flow patterns in the neighborhood of the node, and in the second case regions corresponding to flow patterns in the neighborhood of the focus.

When a focus and a saddle point interact the possible cases are similar to those considered above. If there is a saddle point and a node in the region under consideration (Fig. 7), then there may be a double transition through the speed of sound—first at the node from supersonic to subsonic flow, and then at the saddle point from subsonic to supersonic flow on condition that the separatrix 2 of the saddle point emerges from the node (Fig. 7a). If the separatrix 2 of the saddle point begins in the region Q_6 (Fig. 7b), then a solution with a continuous transition through the speed of sound is possible only along the separatrix 1 of the saddle. In the first case there will be initial values on the straight line $x = 0$ corresponding to flow patterns in the neighborhood of the node and the saddle point, and in the second case the initial data corresponding to solutions with a transition through the velocity of sound fall into region q_6 of the saddle point at the node. Other combinations of singularities may be analyzed in the same manner.

For $\varphi = \varphi(x)$ in the general case there may be any set of singularities in region (2.1) changing their character along the line (2.3). The analysis of their interaction is similar to that outlined above.

3. By way of an example, we give an analysis of the possible flow configurations in a uniform electromagnetic field and a channel which widens linearly.

In the calculations illustrating this point the values of the constants were chosen as follows: $B = 1$, $\varphi = 1$, $\varkappa = 5/3$, the angle between the wall and axis of the channel is $\vartheta^\circ = 20^\circ$ and corresponding to this $y' l^\circ / y_a^\circ = 3.64$, the Mach number and the velocity at the channel exit are $M_a = 1$, $u_a = 0.5$, while $m = 0.32479$.

Assuming that the flow into the inlet of the MHD channel occurs without supplying energy, we find that the velocity $u \leq 1$ everywhere in the flow, and consequently, in the example under consideration, the region of space (x, u, M) is determined by the inequalities

$$0 \leqslant x \leqslant 1, \qquad 0 < u \leqslant 1, \qquad M > 0. \tag{3.1}$$

The singular line (2.3) in the plane $M = 1$, represented in Fig. 8, is composed of two branches; $u = u_1$ and $u = u_2$, corresponding to the plus and minus signs in Eq. (2.3). As the value of the parameter Δ increases the lines of one branch draw closer together, tending to a limiting position for $\Delta \to \infty$:

$$u_1 = \frac{\varphi}{By}, \qquad u_2 = \frac{\varkappa - 1}{\varkappa} \frac{\varphi}{By}.$$

Investigation has shown that along the line $u = u_2$ the singular points are all saddles for all values of Δ and all x in the region (3.1).

In the neighborhood of this branch of the singular line $\alpha < 0$, i.e., the flow occurs in the accelerator configuration.

In the neighborhood of the line $u = u_1$ the flow occurs in the generator configuration. It follows from (2.6) that $\gamma_1 > 0$, $\gamma_2^{**} = 0$ and $\gamma_2 = \gamma_2^* > 0$ for the type of MHD channel under consideration when $\alpha > 0$, and so the singular points may be either foci or nodes.

When $\Delta = 0.1$, the line $u = u_1$ is composed of foci for x and u belonging to region (3.1); when $\Delta = 1$ the singular points are foci for $0 \leq x < 0.44$, and for $0.44 \leq x \leq 1$ the singular points are nodes. For $\Delta \geq 10$ the singular line $u = u_1$ consists of nodes.

Thus in an accelerator corresponding to the example under consideration there exist flow patterns investigated in paragraph 2 of the second case, and in a generator—flow patterns investigated in paragraph 2 in the first and third cases. We shall trace the integral curve with initial data $M_a = 1$, $u_a = 0.5$ and follow its change of character with increase of the parameter Δ. For $x = 1$ the condition $M_b \geq 1$, $p_b \geq p_\infty$ is fulfilled.

Figure 9 gives the variation of the Mach number M along the channel for various Δ. For $\Delta \leq 0.1$ the flow in the channel is entirely supersonic; for $0.1 < \Delta \leq 0.55$ the transition from supersonic to subsonic flow occurs in a shock wave, since the integral curve approached a focus type singularity; for $\Delta \geq 0.55$ the supersonic flow passes continuously to subsonic flow at a node type singularity.

We note in conclusion that the above investigation may be transferred to the more general case of flow with friction and heat exchange, if the corresponding coefficients are known functions of the flow parameters, the longitudinal coordinate, etc.

REFERENCES

1. A. N. Kraiko and F. A. Slobodkin, "Toward a solution of variational problems of one-dimensional magnetohydrodynamics," PMM, vol. 29, no. 2, 1965.

2. E. Resler and W. Sears, "The prospects for magnetoaerodynamics," J. Aeronaut. Soc., vol. 25, no. 4, 1958.

3. F. E. C. Culik, "Compressible magnetogasdynamic channel flow," Z. angewandte Math. und Phys., vol. 15, no. 2, 1964.

4. V. V. Nemytskii and V. V. Stepenov, Qualitative Theory of Differential Equations [in Russian], Gostekhizdat, 1949.

18 January 1966 Moscow